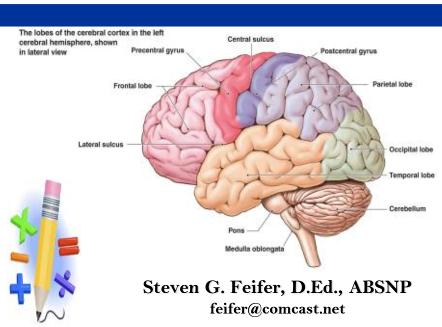
The Neuropsychology of Mathematics:
Developing Evidenced Based Math Interventions



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1

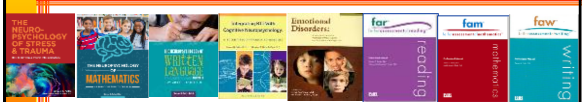
Course Outline

- Six part webinar series on reading, writing, & math disabilities sponsored by Jack Hirose & Associates.
- Introduce a brain-based educational model of dyslexia, dysgraphia, and dyscalculia and classify each disability into distinct subtypes.
- Discuss targeted interventions for all students with academic learning issues.
- Introduce the concept of diagnostic achievement tests versus traditional achievement tests.
- Questions and Comments: **feifer@comcast.net**

2

2

Dr. Feifer's Journey 1993- present



- School psychologist 20+ years
- Diplomate in school neuropsychology
- 2008 Maryland School Psychologist of the Year
- 2009 National School Psychologist of the Year
- Author: 8 books on learning and emotional disorders
- Test Author: FAR & FAM (FAW coming soon)
- Currently in private practice at Monocacy Neurodevelopmental Center in Maryland
  www.schoolneuropsychpress.com

3

3

Presentation Goals

(1) Discuss the international trends in math, and reasons why the United States and Canada lags behind other industrialized nations in mathematics.

(2) Explore the role of various cognitive constructs including working memory, visual-spatial functioning, language, and executive functioning, with respect to math problem solving ability.

(3) Discuss <u>three</u> subtypes of math disabilities, and specific remediation strategies for each type.

(4) Discuss the main neural pathways that contribute to the development of number sense and quantitative reasoning.

(5) Introduce the <u>FAM</u>, a diagnostic test of mathematics designed to examine the underlying cognitive processes that support the acquisition of proficient math skills.

4

2019 NAEP DATA: Mathematics

https://www.nationsreportcard.gov/mathematics?grade=4

- **Survey taken every 2 years by NAEP to capture trends in learning.**
- **Students with disabilities (accommodations allowed), ESL, and private school students included.**

<u>**Content items:**</u> digitally based assessments for grades 4 & 8

a)Number properties and operations – includes computation and understanding of number concepts.

b)Measurement - assesses use of instruments, application of processes, and concepts of area and volume.

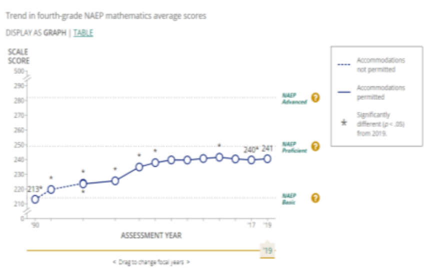
c)Geometry – measures students' knowledge and understanding of shapes in two and three dimensions, spatial reasoning, and geometric properties.

d)Data analysis, statistics, and probability - includes graphical displays and statistics.

e)Algebra – measures students' understanding of patterns, using variables, algebraic representation, functions, and relationships.

5

2019 NAEP DATA: 4th grade



➤ **40% of 4th graders <u>at or above</u> a Proficient level in mathematics (n=149,500 students from 8,280 schools)**

6

2019 NAEP DATA: 4th grade

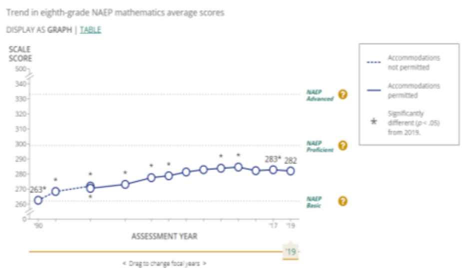
State by State Comparison of Scores

Highest States	>Proficient	Lowest States	>Proficient
Minnesota	53%	Alabama	28%
Massachusetts	50%	New Mexico	29%
New Jersey	48%	Louisiana	29%
Virginia	48%	West Virginia	30%
Wyoming	48%	Alaska	33%
Florida	48%	Arkansas	33%
Pennsylvania	47%	California	34%
Indiana	47%	Washington DC	34%
Utah	46%	Nevada	34%

- National 4th grade average: 241(40% Proficient or Above)

7

2019 NAEP DATA: 8th grade



➢ 33% of 8th graders at or above a Proficient level in mathematics (n=147,400 inclusive of 6,960 schools)

8

2019 NAEP DATA: 8th grade

State by State Comparison of Scores

Highest States	>Proficient	Lowest States	>Proficient
Massachusetts	47%	New Mexico	21%
Minnesota	44%	Alabama	21%
New Jersey	44%	Washington DC	23%
Wisconsin	41%	Louisiana	23%
Washington	40%	West Virginia	24%
South Dakota	39%	Mississippi	24%
Connecticut	39%	Oklahoma	26%
Pennsylvania	39%	Nevada	26%
New Hampshire	38%	Arkansas	27%
Vermont	38%	Hawaii	28%

* National 8th grade average: 282 (33% Proficient or Above)
* Side Note: Approximately **20 percent** of public-school teachers surveyed in both grades reported that a lack of adequate instructional resources was a *moderate* or *serious problem* in 2019.

9

Why Such Low Scores?

- Downward extension of the curriculum.
- Lack of consistency in how math is taught across districts nationwide.
- Teacher training between elementary and high school level.
- Lack of interventions in mathematics as opposed to reading interventions.
- Block scheduling limitations.
- The AP dilemma?? We teach to the test and not for mastery.

10

10

PISA DATA (2018): 15 yr. olds

Program for International Student Assessment)

- A test of **mathematical literacy** for ***15*** year old students which focuses upon the direct application of mathematical principles. The test is administered every three years.
- Approximately 600,000 students completed the assessment in 2018, representing **78** participating countries and approximately **32 million** 15 year-olds .
- The test was ***not*** designed to measure curricular outcomes, but rather to assess mathematics' literacy within a real world context.

Student Selection:

- PISA international contractors sampled schools in each country according to strict technical standards. For further information, go to http://www.oecd.org/pisa/
- Exclusionary criteria included intellectual and physical disabilities, remoteness of school, or insufficient language experiences.
- Most countries randomly assessed between 4000-8000 students
- **Reading** was a featured subject in 2018.

11

11

PISA DATA (2018): 15 yr. olds

Data Released: December 3rd, 2019

Country	Average Score
International Average	489
1. B-S-J-Z China (Beijing, Shanghai, Jiangsu, and Zhejiang)	591
2. Singapore	569
3. Macao-China	558
4. Hong Kong (China)	551
5. Chinese Taipai	531
6. Japan	527
7. Korea	526
8. Estonia	523
9. Netherlands	519
10. Poland	516
11. Switzerland	515
12. Canada	512
13. Denmark	509
14. Slovenia	509
15. Belgium	508
16. Finland	507
17. Sweden	502
18. United Kingdom	502
19. Norway	501
20. Germany	500

12

12

PISA DATA (2018): 15 yr. olds

Data Released: December 3rd, 2019

Country	Average Score
International Average	489
21. Ireland	500
22. Czech Republic	499
23. Austria	499
24. Latvia	496
25. France	495
26. Iceland	495
27. New Zealand	494
28. Portugal	492
29. Australia	491
30. Russia	488
31. Italy	487
32. Slovak Republic	486
33. Luxembourg	483
34. Spain	481
35. Lithuania	481
36. Hungary	481

13

13

PISA DATA (2018): 15 yr. olds

Data Released: December 3rd, 2019

Country	Average Score
International Average	489
37. UNITED STATES	478 (477 in 2015/40th)
38. Belarus	472
39. Malta	472
40. Croatia	464
41. Israel	463
42. Turkey	454
43. Ukraine	453
44. Greece	451
45. Cyprus	451
46. Serbia	448
47. Malaysia	440
48. Albania	437
49. Bulgaria	436
50. United Arab Emirates	435
78. Dominican Republic	325

14

14

4 Reasons for U.S Decline

1. Time on task. Most elementary math instruction occurs in the afternoon, average is 54 instructional minutes per day (89 min language arts).
2. Dry and boring material. Mathematical skill building needs to be FUN, and therefore needs to be presented in the format of games and activities.
3. Too much focus on procedural knowledge. In order to develop conceptual understanding, students should practice multiple methods of problem solving from both a visual-spatial and verbal approach.
4. Teacher Training. Nearly half of elementary education majors have difficulty themselves with a variety of basic math skills (Murphy et al., 2011).

15

15

PISA DATA: Canadian Decline Continues
(2015 = 516……. 2018= 512)

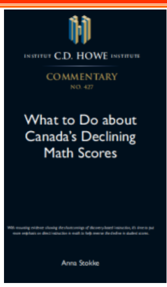
PISA 2003-2012 – Results in paper-based mathematics – Canada and provinces

	2003		2006		2009		2012	
	Average	S.E.	Average	S.E. with linking error*	Average	S.E. with linking error*	Average	S.E. with linking error*
Canada	532	1.8	527	2.4	527	2.6	**518**	2.7
Newfoundland and Labrador	517	2.5	**507**	2.8	**503**	3.4	**490**	4.2
Prince Edward Island	500	2	501	2.7	**487**	3.0	**479**	3.2
Nova Scotia	515	2.2	**506**	2.7	512	3.0	**497**	4.5
New Brunswick	512	1.8	506	2.5	**504**	3.0	**502**	3.2
Quebec	537	4.7	540	4.4	543	3.9	536	3.9
Ontario	530	3.6	526	3.9	526	3.8	**514**	4.5
Manitoba	528	3.1	521	3.6	**501**	4.1	**492**	3.5
Saskatchewan	516	3.9	507	3.6	506	3.8	506	3.6
Alberta	549	4.3	**530**	4.0	**529**	4.8	**517**	5.0
British Columbia	538	2.4	**523**	4.6	**523**	5.0	**522**	4.8

Results in bold indicate a statistically significant difference compared with the baseline (2003).
* The standard error of measurement includes a linking error to account for the comparison of results over time between the baseline (2003) and subsequent years.

16

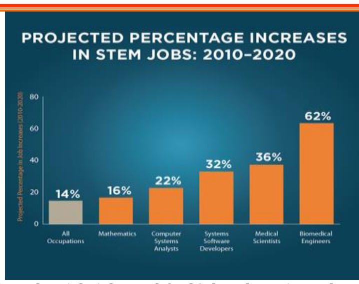
Anna Stoke (May, 2015)
Associate Professor of Mathematics and Statistics at University of Winnipeg



1. Employ the 80/20 rule for direct instruction to discovery based learning.
2. Reduce multiple strategy approaches that are inefficient and place too much burden on working memory.
3. Important concepts are introduced too late, especially Algebra.
4. Teacher training in mathematics needs to improve.

17

The STEM Initiatives



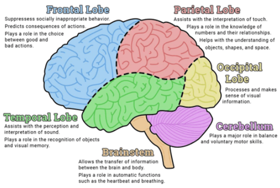
- Given the global demand for high tech workers, there is a greater exportation of jobs overseas due to a combination of cheaper wages, as well as a better educated workforce in mathematics and science.

18

What is School Neuropsychology?

School Neuropsychology: An analysis of learning based upon underlying cognitive processes that support specific academic skills. Since the brain is the seat of ALL learning and behavior, knowledge of cerebral organization should be the key to unlocking the mystery behind most academic endeavors.

The Human Brain



19

19

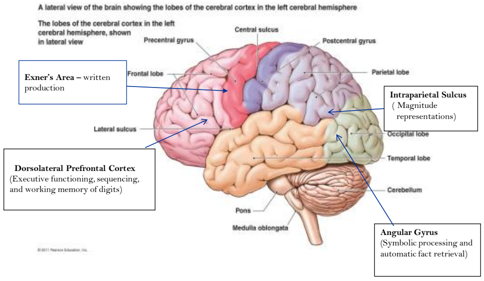
Math Constructs and the Brain

Psychological Construct	Brain Region
Language	Temporal Lobes
Visual-Motor Skills	Exner's Area & Cerebellum
Visual-Spatial Skills	Inferior Parietal Sulcus
Memory (Spatial & Symbolic)	Right Parietal/Left Temporal
Math Fact Retrieval	Angular Gyrus
Attention	Anterior Cingulate
Executive Function & Symbolic Reasoning	Dorsolateral Prefrontal

20

20

Mapping the Math Brain



21

21

What is a Math Disability?

***Dyscalculia** – children with specific math-related deficits including:

a) Learning and retrieving mathematical facts
(Language Retrieval)

b) Executing math calculation procedures
(Symbolic Working Memory)

c) Basic number sense and concept development
(Executive Functioning & Symbolic Logic)

d) Visualizing magnitude representations.
(Visual-Spatial Memory)

Math Learning Disability (MLD) – a generic term referring to children whose math performance in the classroom is substantially below age- and grade-level expectations. Often used when there is unexpected underachievement.

- Up to **20%** of school age children have MLD or persistent difficulty with math (Iuculano et al., 2015)

22

22

The "MLD" Profile

(Geary, 2011: Rasanen, et al., 2009)

1. **Are slower in basic numeric processing tasks:**
   - **Rapidly identifying numbers.**
   - **Making comparisons between magnitude of numbers.**
   - **Counting forwards and backwards**

2. **Struggle in determining quantitative meaning of numbers:**
   - **Poor use of strategies.**
   - **Do not visualize numbers well.**

3. **Have difficulty learning basic calculation procedures needed to problem solve.**

23

23

The "MLD" Profile

(Geary, 2011: Rasanen, et al., 2009)

MLD Error Profile:

- **Prone to procedural errors such as saying "5,6,7" when solving 5 + 3 = ___**
- **Misalign numbers:** 36 +3 = 66
- **Fail to borrow in a sequential manner:** 83 - 44 = 41
- **Often deploy the wrong computational process:** *"The school store sold twice as many pencils to Sam than Robert. If Sam was sold four pencils, how many pencils were sold to Robert?"* _8_
- **Poor retrieval of basic facts:** 7 X 6 = 35

24

24

Language and Mathematics

<u>Language Skills:</u> (temporal lobes)

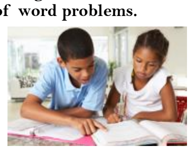
- Most Asian languages have linguistic counting systems past *ten* (*ten-one, ten-two, etc)* whereas English deviates from base-10 system (Campbell & Xue, 2001).
- In English counting system, decades come first then unit *(i.e. twenty-one)* or sometimes this is reversed *(i.e. fifteen, sixteen, etc…)*
- Chinese numbers are brief *(i.e. 4=si, 7=qi)* allowing for more efficient memory. By age four, Chinese students can count to 40, U.S. students to 15.
- U.S. kids spend <u>180</u> days in school
  South Korea children spend <u>220</u> days in school
  Japanese children spend <u>243</u> days in school

25

Language and Mathematics

<u>Language Skills:</u> (temporal lobes)

- Early math skills tend to be verbally encoded, and that is how we initially learn math facts.
- Children with math disabilities frequently have delays in their language development (Shalev et al., 2000)
- Word problems offer an intricate relationship between language and mathematics. Terms such as *all, some, neither, sum, etc.* may be confusing when embedded in the grammatical complexity of word problems.



26

Working Memory and Math

- <u>Phonological Loop</u> - holds and manipulates acoustic information. Housed in *left temporal lobes* and plays a role in <u>holding and manipulating</u> sounds through verbal rehearsal.
- <u>Visual-Spatial Sketchpad</u> - holds visual, spatial, and kinesthetic information in temporary storage by way of mental imagery. Housed along inferior portions of *right parietal lobes* and plays a role in visualizing magnitudes and amounts.
- <u>Episodic Buffer</u> – a temporary storage system integrating both phonological and visual-spatial information. Important in the mental <u>manipulation</u> of digits.
- <u>Central Executive System</u> – coordinates working memory systems and allocates attention resources. Impacted by <u>anxiety</u> and <u>emotional distress</u>!!!

27

Working Memory and Math

Working Memory System	Mathematical Skill
• Phonological Loop	• Retrieval of math facts • Writing dictated numbers
• Visual-Spatial Sketchpad	• Magnitude comparisons • Geometric Proofs
• Episodic Buffer	• Mental math with symbols
• Central Executive System	• Inhibiting distracting thoughts • Modulating anxiety • Regulating system.

28

Horizontal Vs. Vertical
(Trbovich & LeFevre, 2003)

- Solving problems in a vertical format required the use of more visual resources, particularly the visual-spatial sketchpad of working memory.
- Solving problems in a horizontal format required more phonological resources resulting in ***slower*** performance.

A 32 + 6	B 6 + 32
C 32 + 6	D 6 + 32

29

Interventions for Lower Working Memory

- Number-line situated on student's desk.
- Use a calculator.
- Increase number sense through games such as dice, domino's, cards, etc..
- Encourage paper and pencil use while calculating equations.
- Use mnemonic techniques to teach math algorithm's and sequential steps to problem solving (i.e. The steps for long division are
  Divide, Multiply, Subtract, Bring Down:
  Dad Mom Sister Brother
  Dead Monkies Smell Bad
- Reduce anxiety in the classroom.

30

The Truth About Math Anxiety: Do We Have a Math Phobia?

* According to the Office of Economic Development , 59% of 15 yr olds olds taking the PISA Test often worry about math, and 30% feel helpless when executing a problem.

- Teachers who have math anxiety tend to impact girls' math performance more than boys (Beilock et al., 2010). Similar findings with mothers and daughter dyads (Casad et al., 2015).
- Glaring weakness in research is which type of math skill impacted most by anxiety (Dowker et al., 2016).

31

31

Math Anxiety

<u>Cortisol</u> – a glucocorticoid (glucose-cortex-steroid) that regulates the metabolism of glucose in the brain. A balance or homeostasis of cortisol is needed for optimal brain functioning. Too much *(Cushing's Syndrome)*…too little *(Addison's Disease)*.

- Anxiety impacts cognition and learning by way of <u>working memory</u> (Dowker et al., 2015)
- Anxiety and stress alters amygdala to PFC connections leading to poor <u>executive functioning</u> (Berens et al., 2017).

32

32

The Truth About Math Anxiety: Do We Have a Math Phobia?

<u>Implicit Messages</u>:

"Oh not to worry Billy, I was never that good in math either."
"Wow, are you taking Algebra II….that is sooooo hard!"
"Hey Ritchie…it doesn't matter if you do not understand your math homework, you will never use this stuff in real life."

<u>CAUSES OF MATH ANXIETY:</u>

- Timed tests
- Pop quizzes
- Being called upon to write a math problem on the board
- Speeded skill drills and classroom competitions
- Teaching too quickly before concepts are consolidated
- Unit tests that cover too much information
- No visual cues
- Poor instruction
- Classroom climates that prevent students from asking questions
- Stressing teacher's own algorithm

33

33

Executive Functioning and Math

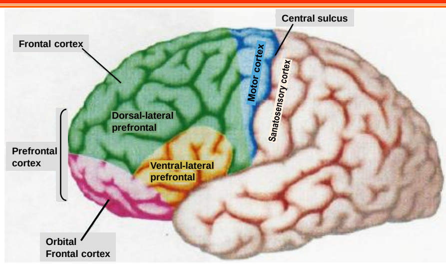
Executive Functioning Skills: (frontal lobes)

- Executive control mechanisms are a set of directive processes such as planning, self-monitoring, organizing, and allocating attention resources to effectively execute a goal directed task.
- Executive functioning dictates "*what to do when*", a critical process in solving word problems.
- Executive functioning allows students to choose an appropriate algorithm when problem solving.

34

34

Executive Functioning and Math



35

35

The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• Selective Attention	• *Anterior Cingulate/* • *Subcortical structures*	• Poor attention to math operational signs • Place value mis-aligned
• Planning Skills	• *Dorsal-lateral PFC*	• Selection of math process impaired • Difficulty determining salient information in word problems

36

36

Executive Functioning and Math

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• Organization Skills	• *Dorsal-lateral PFC*	• Inconsistent lining up math equations • Frequent erasers • Difficulty setting up problems
• Self-Monitoring	• *Dorsal-lateral PFC*	• Limited double-checking of work • Unaware of plausibility to a response.
• Cues Pattern Recognition	• *Dorsal-lateral PFC*	• Symbolic reasoning

37

37

Mathematical Proficiency

1. **Conceptual Understanding –** an overall comprehension of math concepts**.**
2. **Procedural Fluency –** efficiency and flexibility in carrying out math procedures
3. **Strategic Competence –** ability to formulate, represent, and solve math problems.
4. **Adaptive Reasoning –** capacity for logical thought, reflection, and explanation.
5. **Productive Disposition –** ability to see math as being a useful and worthwhile endeavor and belief in one's own efficiency.

38

Building a Math Brain: 4 Neurocognitive Factors

1. **Approximate Number System - non-symbolic representation of math represented by space and time.**
2. **Connectivity - linking non-symbolic representations with symbolic representations (numerals) to form our own internal number line.**
3. **Automaticity - facts and procedures.**
4. **Quantitative knowledge - mathematical reasoning emerges from the development of *number-sense* as students learn to apply mathematics to real world problems.**

39

39

Let's Stay Connected!

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Tests: FAR- 2015 FAM- 2016 FAW - 2020
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40